FONCTIONS CIRCULAIRES ET CIRCULAIRES RECIPROQUES

I – Fonctions circulaires directes : Formulaire.

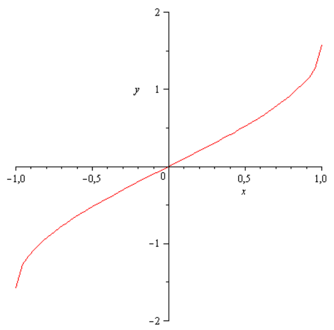
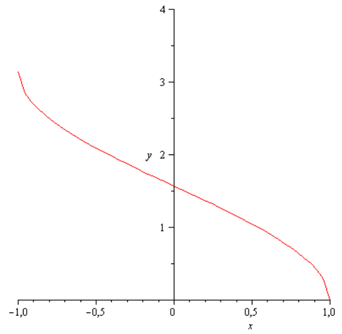
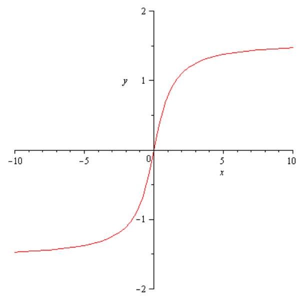
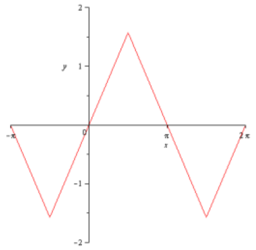
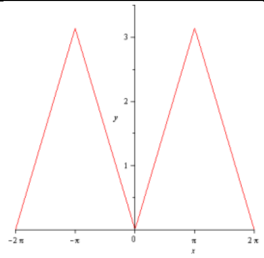
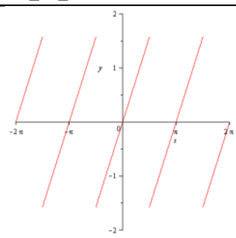
$\sin' x = \cos x$; $\cos' x = -\sin x$; $\tan' x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$; $\cot\text{an}' x = -1 - \cot\text{an}^2 x = \frac{-1}{\sin^2 x}$

$\begin{cases} \sin(a+b) = \sin a \cos b + \cos a \sin b \\ \cos(a+b) = \cos a \cos b - \sin a \sin b \end{cases}$; et $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

$\begin{cases} \sin 2x = 2 \sin x \cos x \\ \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \end{cases}$ et $\begin{cases} \cos^2 x = \frac{1+\cos 2x}{2} \\ \sin^2 x = \frac{1-\cos 2x}{2} \end{cases}$

En posant $\boldsymbol{t = \tan\frac{x}{2}}$ on a : $\sin x = \frac{2t}{1+t^2}$; $\cos x = \frac{1-t^2}{1+t^2}$; $\tan x = \frac{2t}{1-t^2}$

II – Fonctions circulaires réciproques : Formulaire.

Arcsin	Arccos	Arctan
$Arcsin : [-1;1] \to \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$	$Arccos : [-1;1] \to [0;\pi]$	$Arctan : \mathbb{R} \to \left]-\frac{\pi}{2};\frac{\pi}{2}\right[$
$Sur\ ]-1;1[:$ $Arcsin' x = \frac{1}{\sqrt{1-x^2}}$	$Sur\ ]-1;1[:$ $Arccos' x = \frac{-1}{\sqrt{1-x^2}}$	$Sur\ \mathbb{R} :$ $Arctan' x = \frac{1}{1+x^2}$
		
$\forall x \in [-1;1] : \sin(Arcsin\, x) = x$	$\forall x \in [-1;1] : \cos(Arccos\, x) = x$	$\forall x \in \mathbb{R} : \tan(Arctan\, x) = x$
$f : x \to Arcsin(\sin x)$ f est 2π-périodique, impaire et $Arcsin(\sin x) = \begin{cases} x & si\ x \in \left[0;\frac{\pi}{2}\right] \\ \pi - x & si\ x \in \left[\frac{\pi}{2};\pi\right] \end{cases}$ 	$g : x \to Arccos(\cos x)$ g est 2π-périodique, paire et $\forall x \in [0;\pi] : Arccos(\cos x) = x$ 	$h : x \to Arctan(\tan x)$ h est définie sur $\mathbb{R} / \left(\frac{\pi}{2} + k\pi\right)$ π-périodique, impaire et $\forall x \in \left]-\frac{\pi}{2};\frac{\pi}{2}\right[: Arctan(\tan x) = x$ 

$\forall x \in [-1;1] : Arccos\, x + Arcsin\, x = \frac{\pi}{2}$ et $\forall x \in \mathbb{R}^* : Arctan\, x + Arctan\frac{1}{x} = \frac{\pi}{2} sgn(x)$

$\forall x \in [-1;1] : \sin(Arccos\, x) = \cos(Arcsin) = \sqrt{1-x^2}$

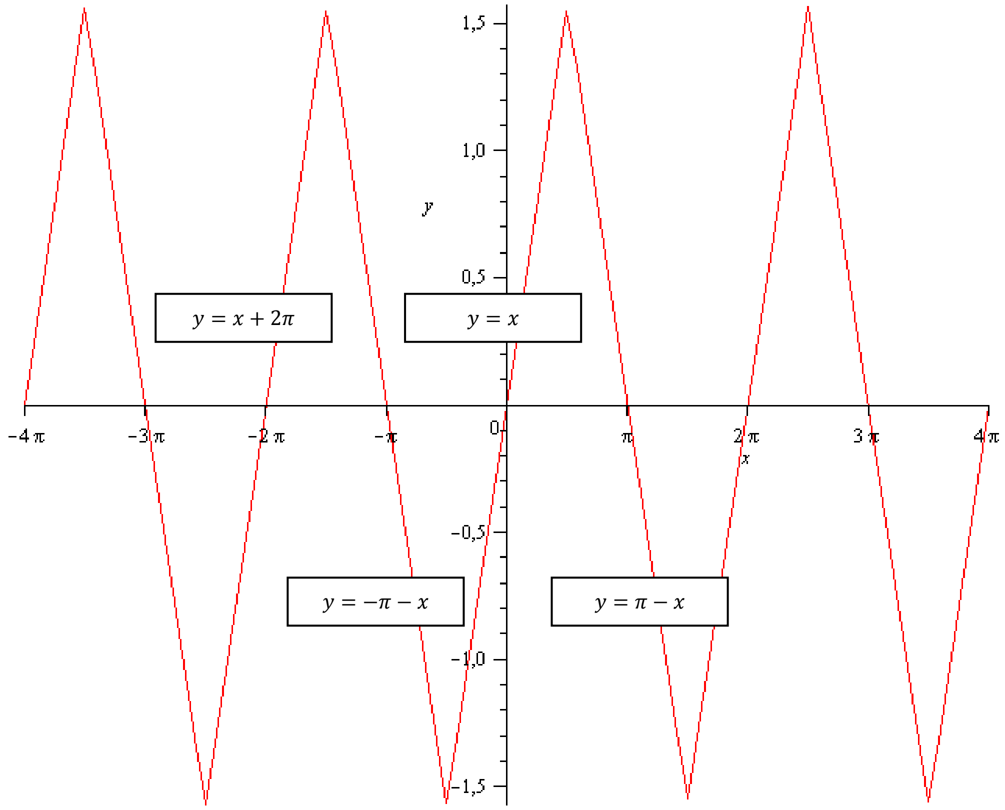
$\forall x \in [-1;1]/\{0\} : \tan(Arccos\, x) = \frac{\sqrt{1-x^2}}{x}$ et $\forall x \in]-1;1[: \tan(Arcsin\, x) = \frac{x}{\sqrt{1-x^2}}$

$\forall x \in \mathbb{R} : \cos(Arctan\, x) = \frac{1}{\sqrt{1+x^2}}$ et $\sin(Arctan\, x) = \frac{x}{\sqrt{1+x^2}}$

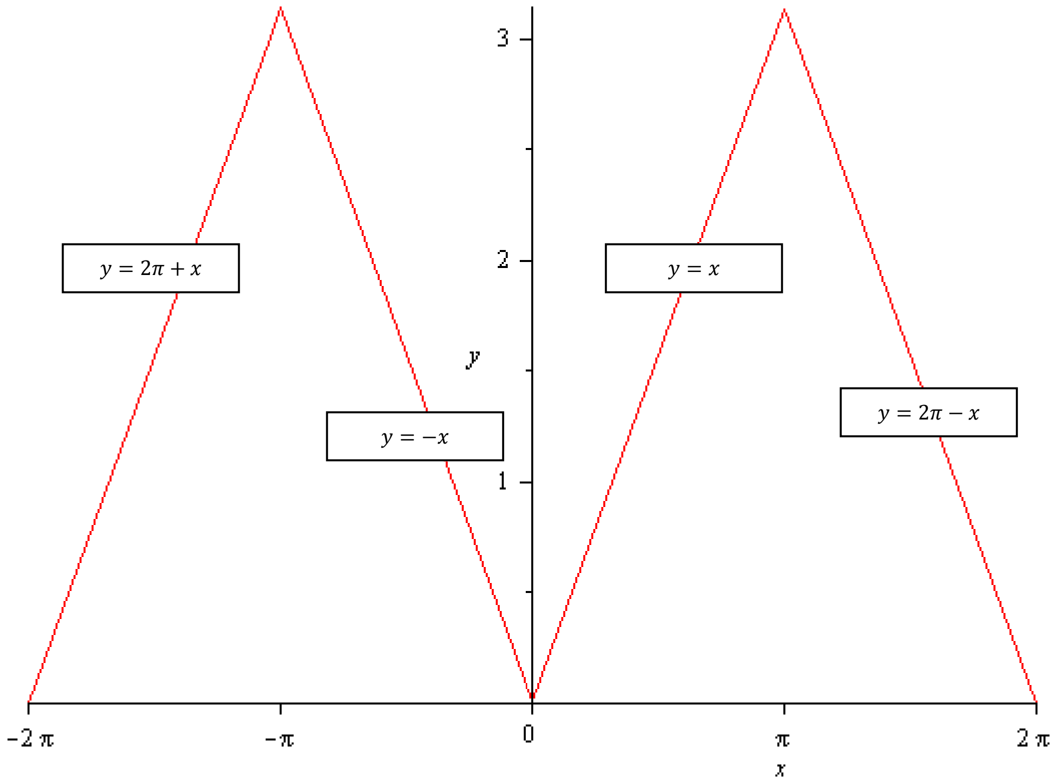
$Arctan\, a + Arctan\, b = \begin{cases} Arctan\frac{a+b}{1-ab} & si\ ab < 1 \\ \frac{\pi}{2} sgn(a) & si\ ab = 1 \\ Arctan\frac{a+b}{1-ab} + \pi\, sgn(a) & si\ ab > 1 \end{cases}$

M. Duffaud : http://www.math93.com/gestclasse/classes/ipsa_sup.html

$f: x \to Arcsin\,(\sin x)$



$g: x \to Arccos\,(\cos x)$



$h: x \to Arctan\,(\tan x)$