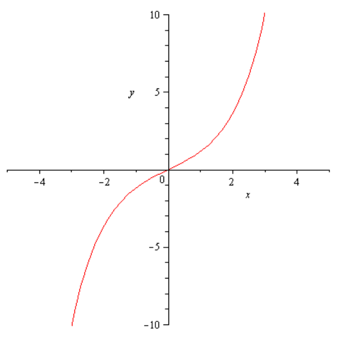
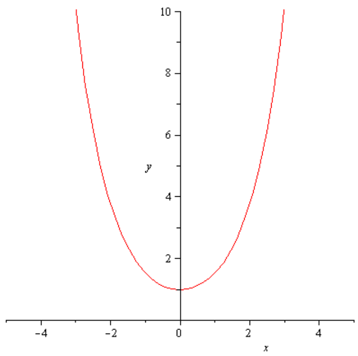
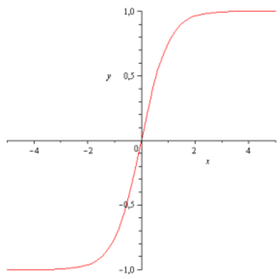
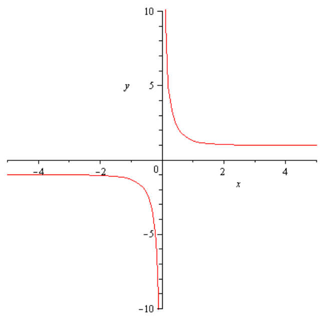
FONCTIONS HYPERBOLIQUES ET HYPERBOLIQUES RECIPROQUES

I – Fonctions HYPERBOLIQUES directes

Sinus hyperbolique $sh : \mathbb{R} \to \mathbb{R}$ Impaire	**Cosinus hyperbolique** $ch : \mathbb{R} \to [1; +\infty[$ Paire	**Tangente hyperbolique** $th : \mathbb{R} \to \,]-1; 1[$ Impaire	**Cotangente hyperbolique** $\coth : \mathbb{R}/\{0\} \to \mathbb{R}$ Impaire
$sh\, x = \dfrac{e^x - e^{-x}}{2}$	$ch\, x = \dfrac{e^x + e^{-x}}{2}$	$th\, x = \dfrac{e^{2x} - 1}{e^{2x} + 1}$	$coth\, x = \dfrac{e^{2x} + 1}{e^{2x} - 1}$
$sh' x = ch$	$ch' x = sh$	$th' x = \dfrac{1}{ch^2 x} = 1 - th^2 x$	$coth' x = \dfrac{-1}{ch^2 x} = 1 - coth^2 x$
			

$\boxed{ch^2 x - sh^2 x = 1}$; $\boxed{chx + shx = e^x}$; $\boxed{chx - shx = e^{-x}}$

Pour obtenir le formulaire de trigonométrie hyperbolique à partir du formulaire de trigonométrie circulaire, il suffit de remplacer : $\boxed{\begin{cases} \cos & \to \quad ch \\ \sin & \to i.sh \\ \tan & \to i.th \end{cases}}$

$\begin{cases} \boxed{sh\,(a+b) = sh\,a\,ch\,b + ch\,a\,sh\,b} \\ \boxed{ch(a+b) = ch\,a\,ch\,b + sh\,a\,sh\,b} \end{cases}$; et $\boxed{th(a+b) = \frac{th\,a + th\,b}{1 + th\,a\,th\,b}}$

$\begin{cases} \boxed{sh\,2x = 2\,sh\,x\,ch\,x} \\ \boxed{ch\,2x = ch^2 x + sh^2 x = 2\,ch^2 x - 1 = 1 + 2sh^2 x} \end{cases}$ et $\boxed{\begin{cases} ch^2 x = \frac{1 + ch\,2x}{2} \\ sh^2 x = \frac{ch\,2x - 1}{2} \end{cases}}$

II – Fonctions HYPERBOLIQUES réciproques

$Argsh : \mathbb{R} \to \mathbb{R}$	$Argch : [1; +\infty[\to [0; +\infty[$	$Argth :]-1; 1[\;\to\; \mathbb{R}$
Argsh est impaire et $\mathcal{C}^\infty$ sur $\mathbb{R}$	Argch est $\mathcal{C}^\infty$ sur]1 ;+∞[	Argth impaire et $\mathcal{C}^\infty$ sur]-1 ;1[
$\boxed{Argsh' x = \dfrac{1}{\sqrt{1+x^2}}}$	$\boxed{Argch' x = \dfrac{1}{\sqrt{1+x^2}}}$	$\boxed{Argth' x = \dfrac{1}{1-x^2}}$

$\boxed{\forall x \in \mathbb{R};\; Argshx = ln\left(x + \sqrt{1+x^2}\right)}$ $\boxed{\forall x \in [1; +\infty[\,;\; Argchx = ln\left(x + \sqrt{x^2 - 1}\right)}$

$$\boxed{\forall x \in \,]-1; 1[\,;\; Argthx = \frac{1}{2} ln\left(\frac{1+x}{1-x}\right)}$$